

Experimental Dissipative Quantum Chaos

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Lucas Sá,
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Sergiy Denisov

Roadmap

- Quantum Chaos and Random Matrix Theory (RMT)
 - Classical vs Quantum Chaos
 - Properties of matrices produced by different stochastic models
 - Connect with theoretical physical model and real experiments

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 - Retrieval protocol to model the dynamics of a quantum system
 - Uses measurement data, Pauli expectation values.
 - New protocol, easily optimised with gradient-based methods.

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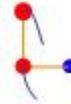
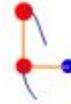
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 - Uses measurement data, Pauli expectation values.
 - New protocol, easily optimised with gradient-based methods.
- Results
 - Model quantum circuit as quantum maps. Retrieve with QPT.
 - Tunable platform, different circuits -> Chaotic/Regular, Open/closed
 - Spectral support of parameterized quantum circuits is captured by *diluted unitaries*
 - We are able to resolve higher order spectral statistics, like *complex spacing ratio*, with our methods. We are able to engineer *integrable* openness in circuits, and detect this. However, for deep circuits, it transitions to chaos because of latent noise

Classical and Quantum Chaos

- Classical Chaos -
Initial perturbation ->
large evolved deviation

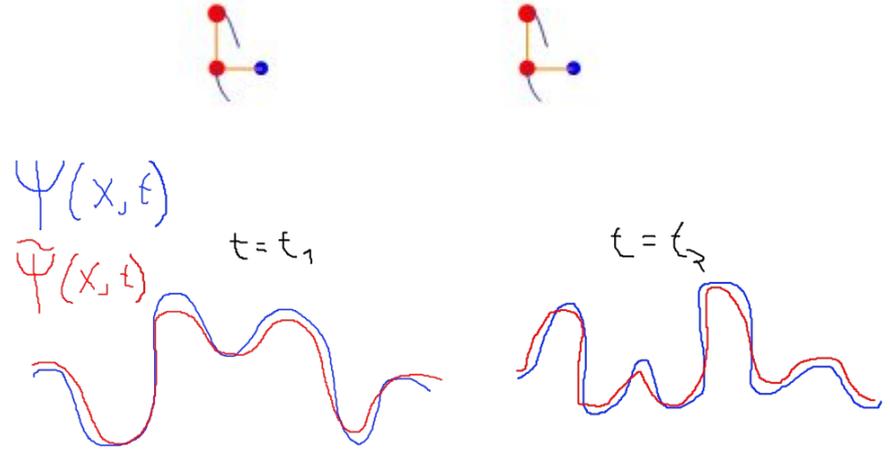
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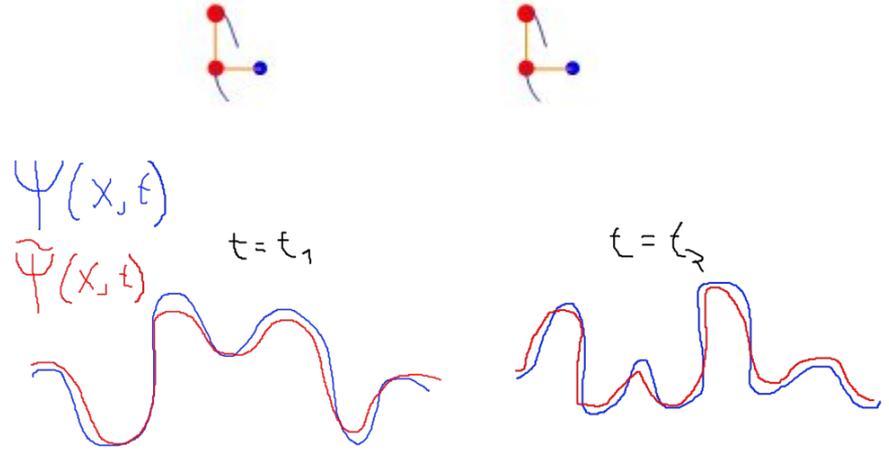
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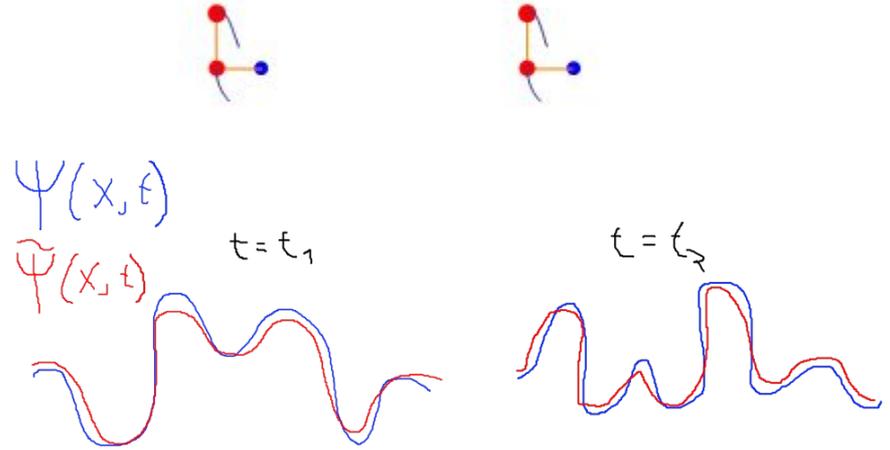
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Chaos enters QM somewhere other than on the levels of amplitudes.

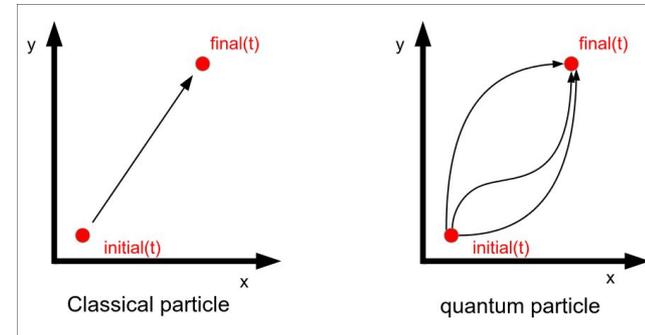


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- Classical phenomenon emerge from quantum mechanics? Chaos enters QM somewhere other than on the levels of amplitudes.
- Many quantum systems have classical limits.



$\hbar \rightarrow 0$

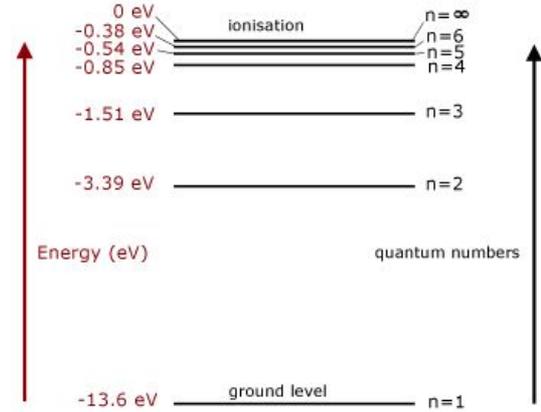


Spectral Chaos

- One way to determine quantum chaos is to look at spectral properties.

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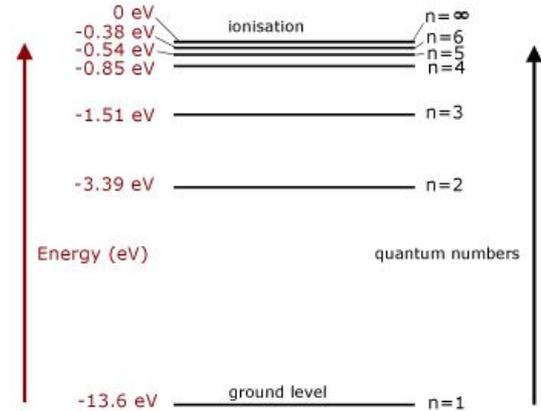
- One way to determine quantum chaos is to look at spectral properties.
- Bohigas–Giannoni–Schmit conjecture: Quantum systems whose classical equivalent is chaotic exhibit energy level spacings that follow Wigner-Dyson distributions.



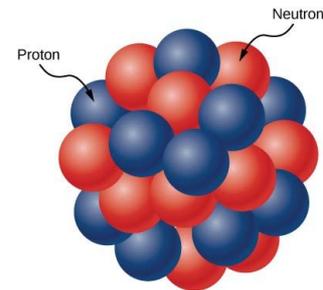
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Spectral Chaos

- One way to determine quantum chaos is to look at spectral properties.
- Bohigas–Giannoni–Schmit conjecture: Quantum systems whose classical equivalent is chaotic exhibit energy level spacings that follow Wigner-Dyson distributions.
- Not all quantum systems have classical equivalents. The tools of quantum chaos are still available!



$$H(\vec{\theta}) \rightarrow s_n = E_{n+1} - E_n \sim pWD(s)$$



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Random Matrix Theory

- Random Matrix Theory investigate (spectral) properties of stochastically generated matrices.
- In general, a spectrum $\{\lambda_n\}$ of a matrix H relates to its elements in a very complicated way.
- Remarkably, can be computed analytically when the elements follow different stochastic models. In particular, the Gaussian Orthonormal Ensemble (GOE).

$$\det(H - \lambda I) = 0$$

$$G_{ij} \sim \mathcal{N}(0, \frac{1}{\sqrt{d}})$$

$$H = \frac{G + G^T}{\sqrt{2}}$$

$$P(\lambda_1, \dots, \lambda_N) = \frac{1}{Z_N} \prod_{i < j} |\lambda_i - \lambda_j| \prod_j e^{-\frac{1}{4}\lambda_j^2} d\lambda_j$$

Random Matrix Theory – Energy Spacing

- With a joint probability for the eigenvalues of a GOE system, the energy spacings can be computed

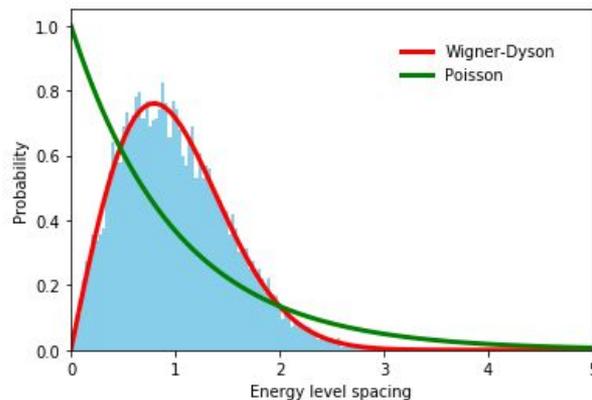
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Random Matrix Theory – Energy Spacing

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- Wigner-Dyson exhibit distinct eigenvalue repulsion, opposed to uncorrelated Poisson statistics

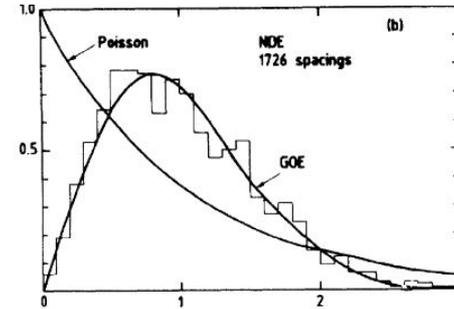
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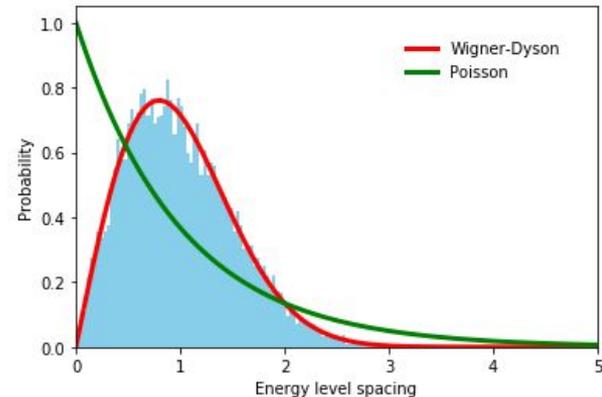
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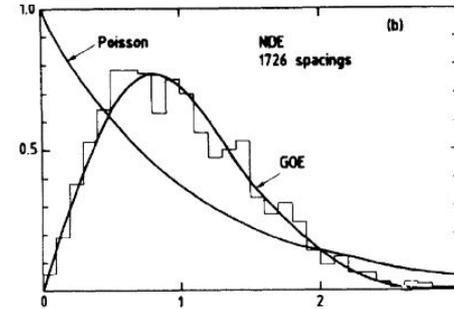
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g.1. Nearest neighbor spacing histogram for : (a) ^{90}Er , (b) Nuclear Data Ensemble (NDE).



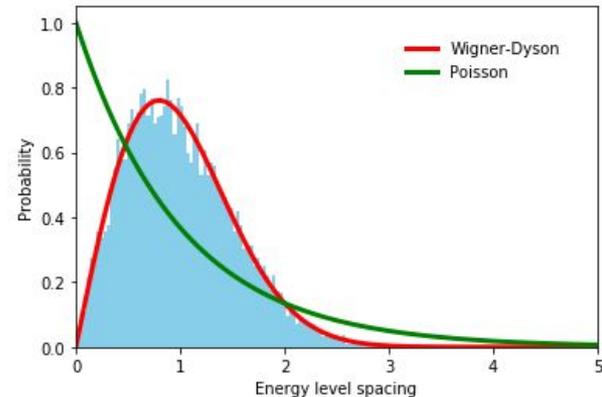
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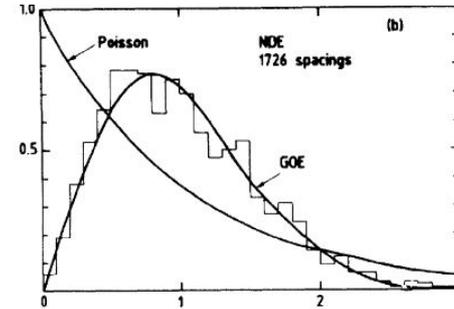
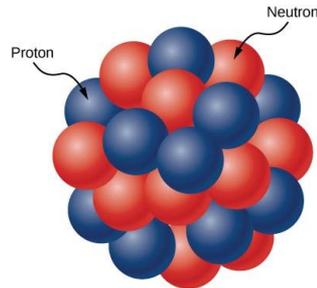
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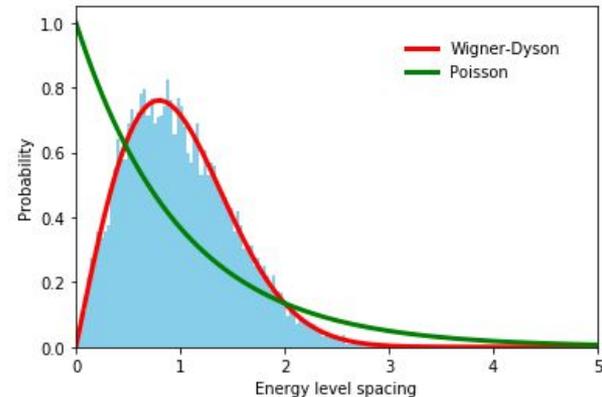
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- Have also checked consistency against theoretical models, like the shell model



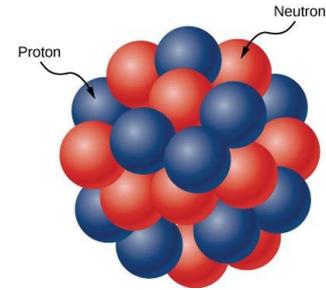
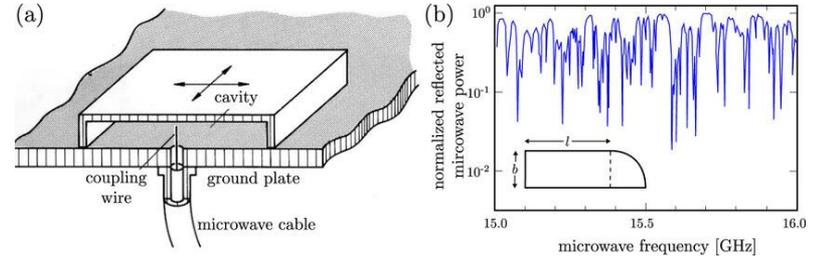
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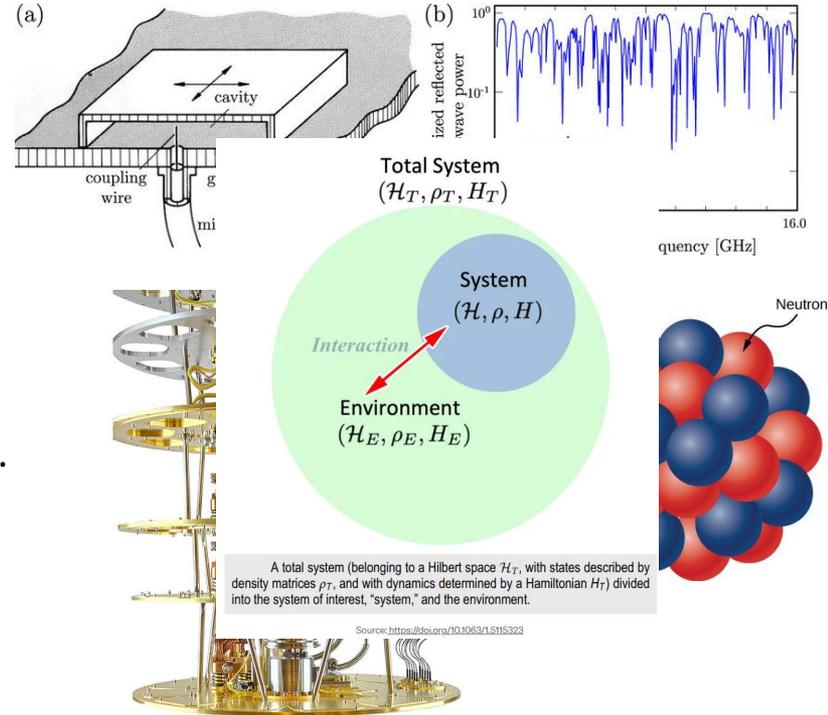
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Natural: nuclei
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Engineered: microwave Billiards and quantum computers.
- What has lacked is experimental results for systems that are dissipative, i.e., influenced by noise and other effects from an environment.



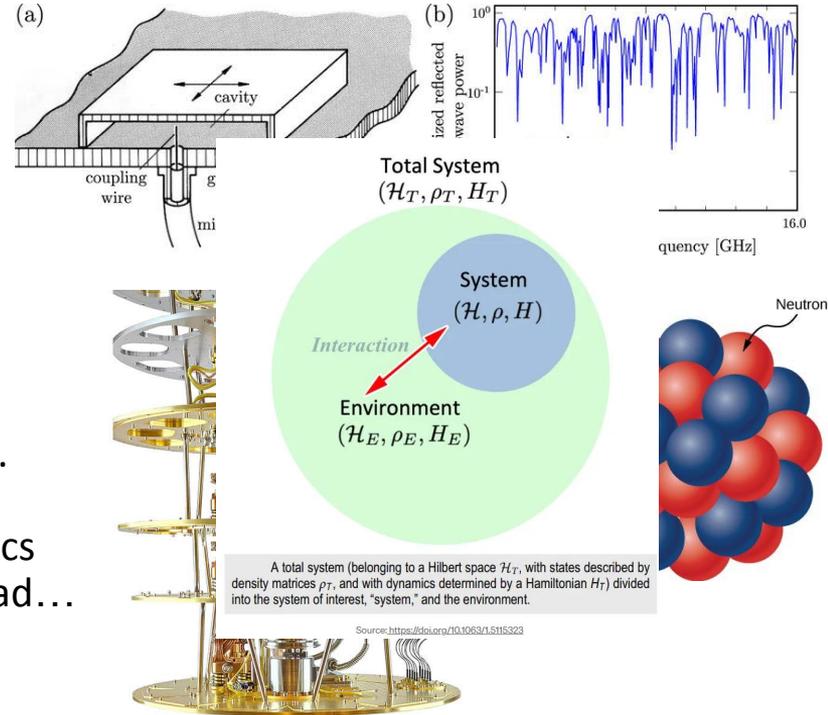
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- What has lacked is experimental results for systems that are dissipative, i.e., influenced by noise and other effects from an environment.
- What systems to investigate? What characteristics to look for? Turned out to be a serendipitous road...



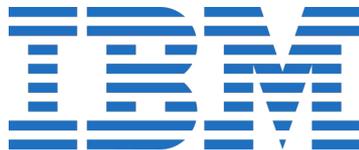
Tempting Idea – Quantum Computers!

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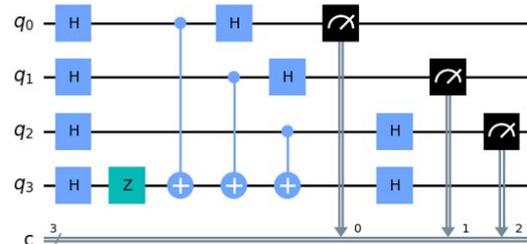
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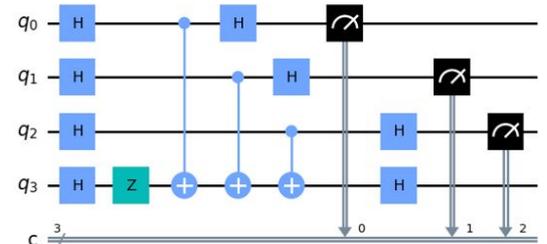
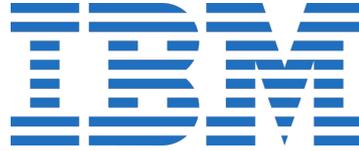
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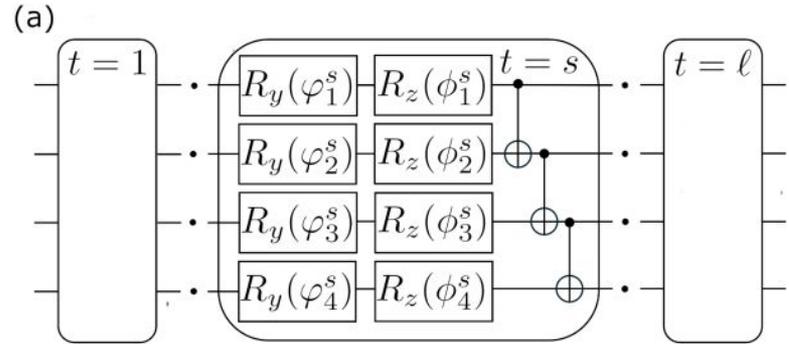
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- Noisy! A curse for some. A blessing for us.



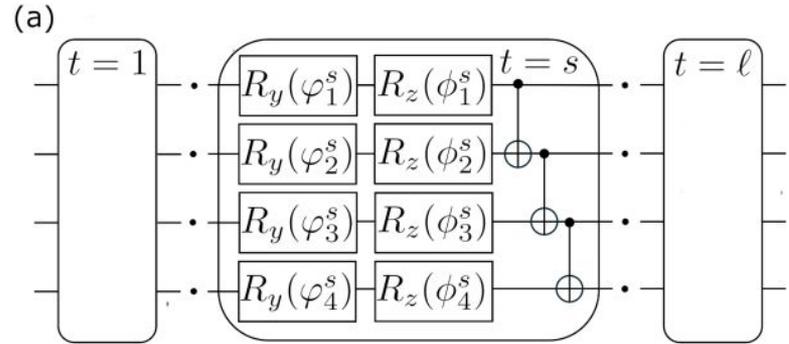
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- Quantum circuit consisting of layers of random rotations and “ladders” of CNOTs



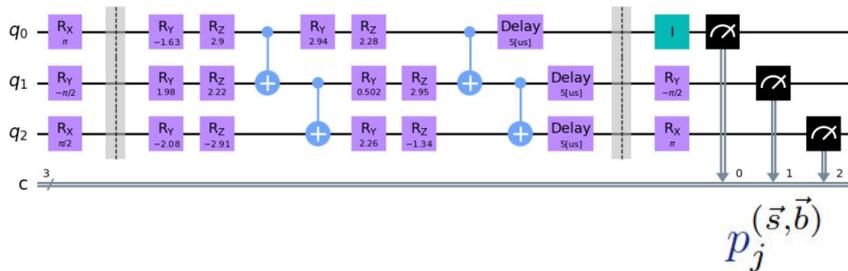
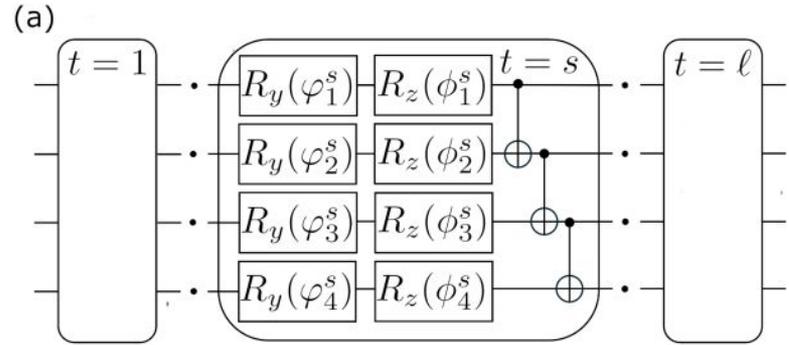
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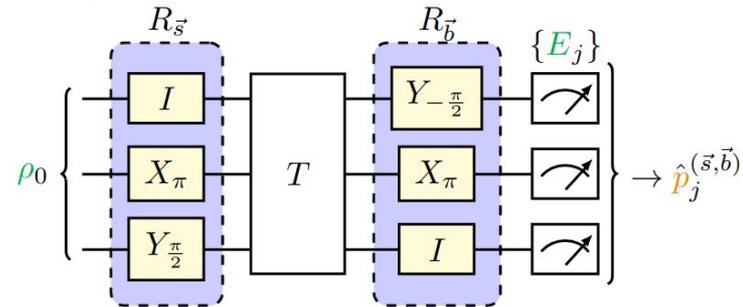
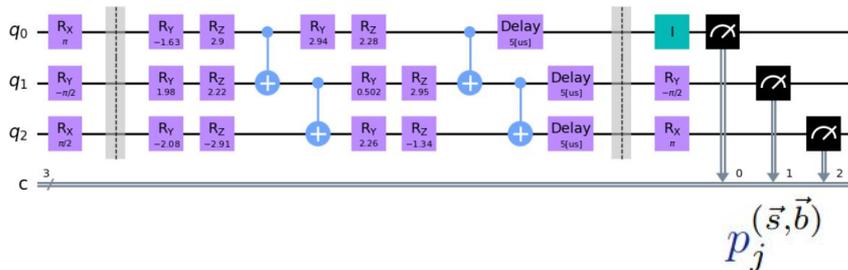
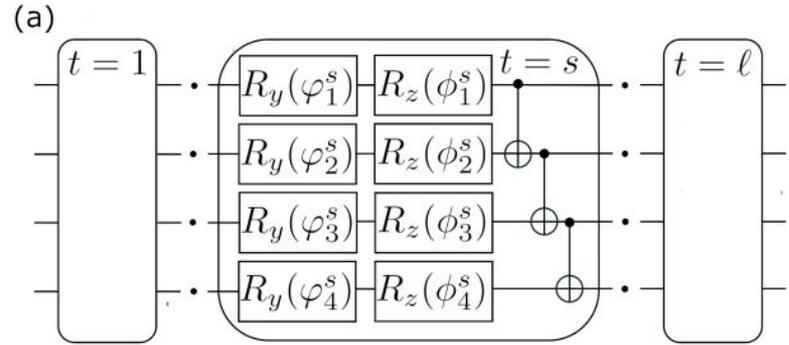
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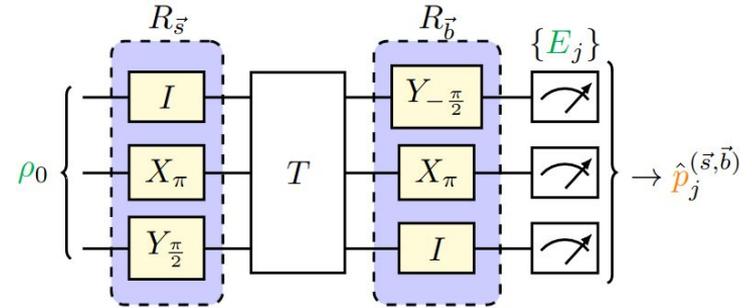
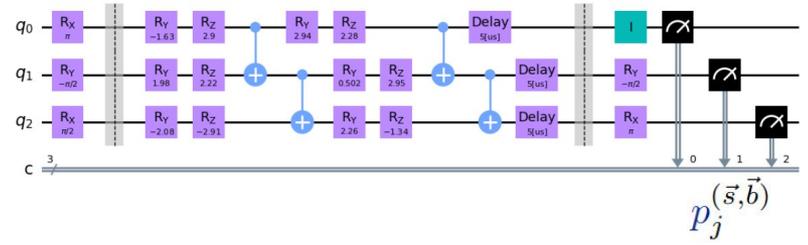
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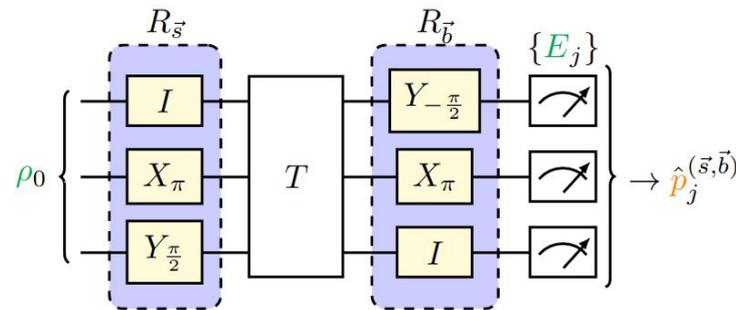
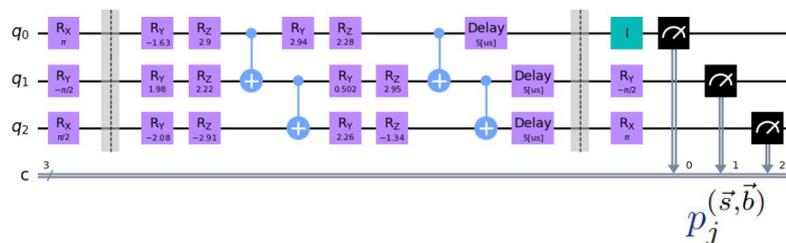
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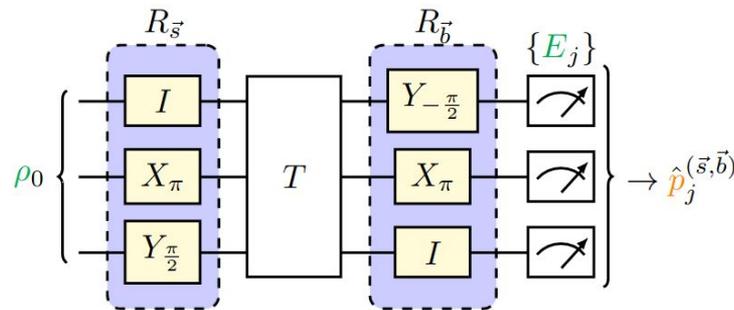
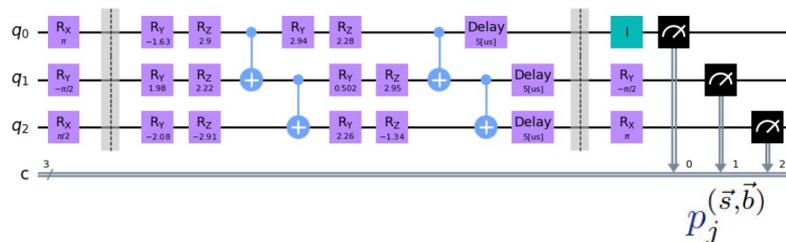
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- Diagonalize T , obtaining the spectrum

$$\Phi_T ||\psi_R^{(i)}\rangle\rangle = \lambda_i ||\psi_R^{(i)}\rangle\rangle$$

$$\{\bar{\lambda}_i\}_i = \{\lambda_i\}_i \quad |\lambda_i| \leq 1$$

$$\lambda_0 = 1 \quad ||\psi_R^{(0)}\rangle\rangle = ||\rho_{ss}\rangle\rangle$$



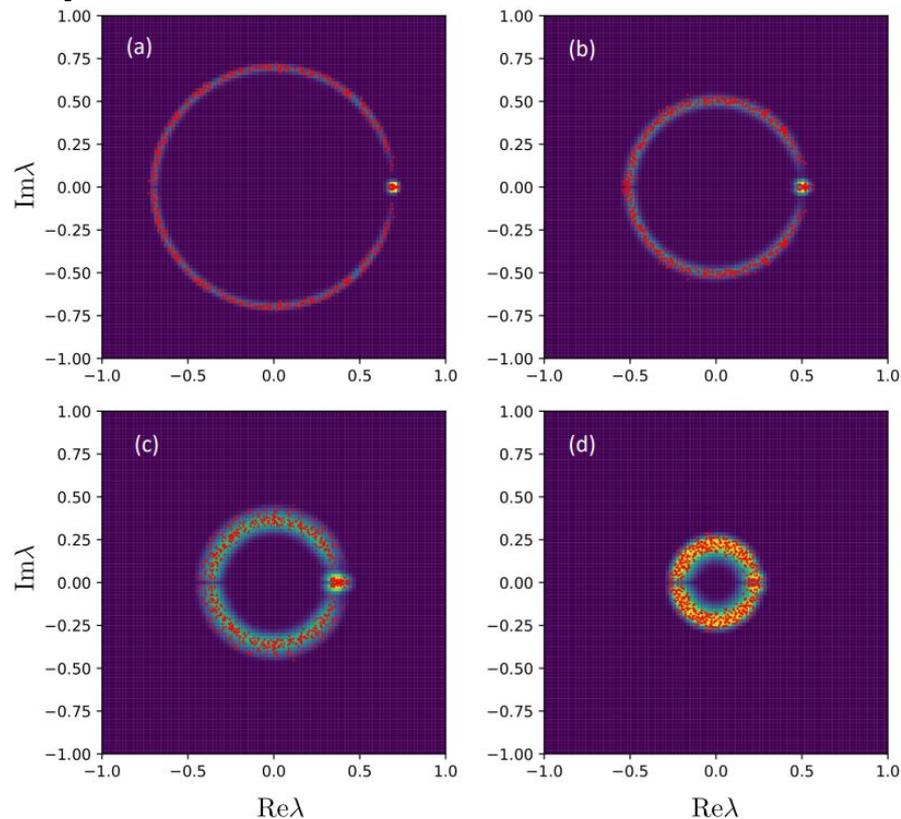
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Spectrum of Retrieved Map

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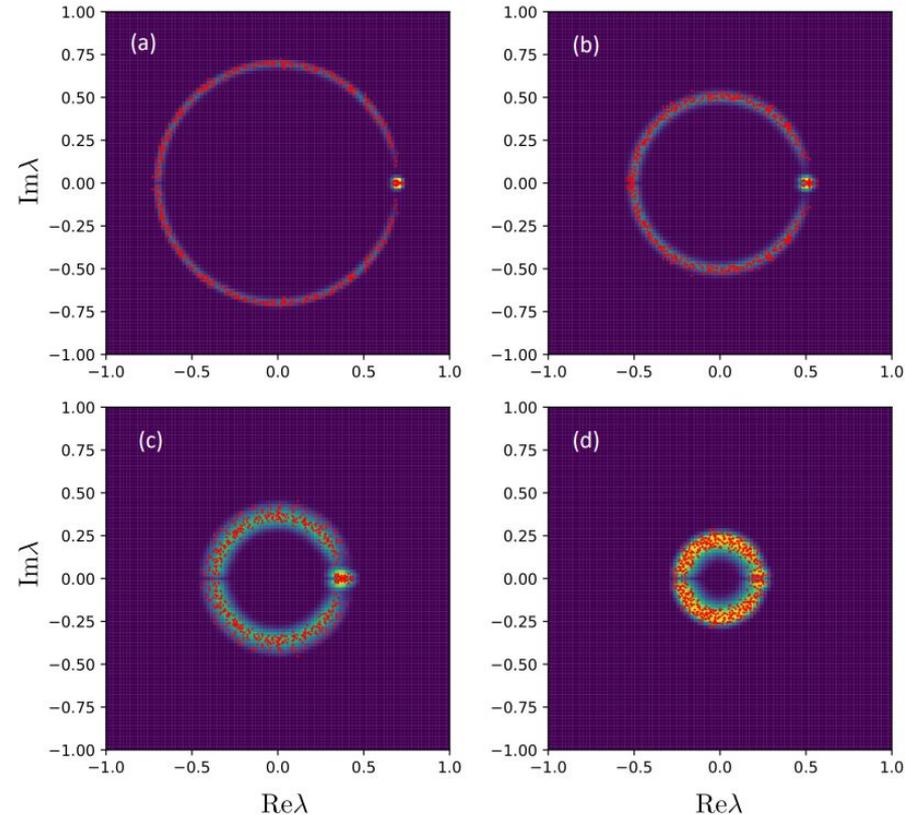
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The eigenvalues distribute in an annulus.
Sharp boundary of an inner and outer radius.
Annulus collapse and broaden as a function of L .



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The eigenvalues distribute in an annulus.
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Annulus collapse and broaden as a function of L .
- Very particular form! Do we know of any theoretical models capturing this behaviour?



Diluted Unitaries

Sá. et. al. *Spectral transitions and universal steady states in random Kraus maps and circuits*. Phys. Rev. B 102, 134310 (2020)

- A two-parameter stochastic ensemble of quantum channels.

$$T_{DU}(\rho) = (1 - p)U\rho U^\dagger + p \sum_{i=1}^r K_i \rho K_i^\dagger$$

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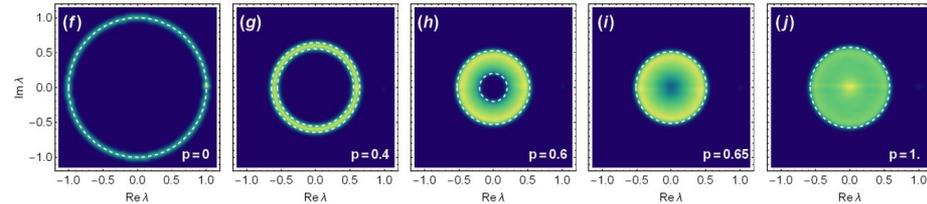
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- Also produce an annulus shaped spectral support.

$$T_{DU}(\rho) = (1 - p)U\rho U^\dagger + p \sum_{i=1}^r K_i \rho K_i^\dagger$$

$$R_{\pm} = \frac{1}{\sqrt{d}} \sqrt{(1-p)^2 d \pm p^2}$$



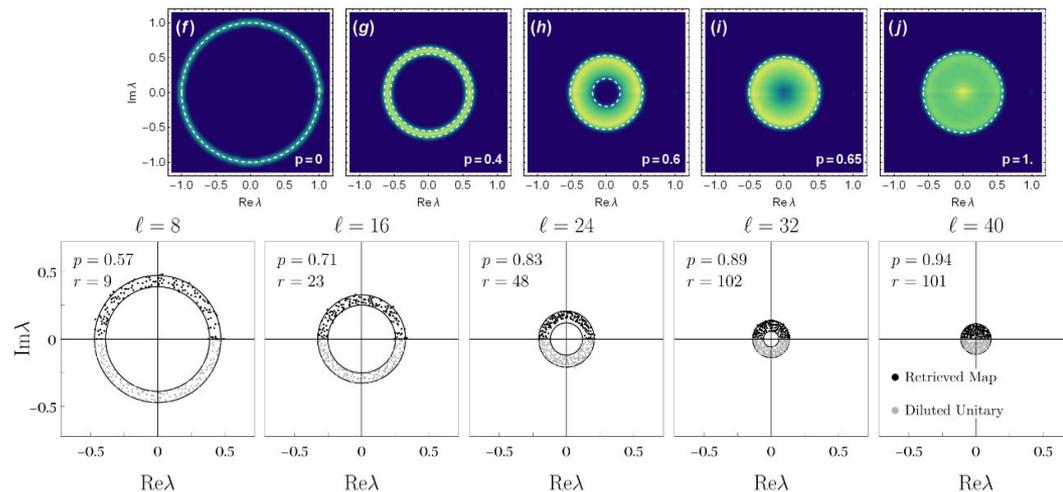
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- Also produce an annulus shaped spectral support.
- Inner and outer radius is related to \mathbf{p} and \mathbf{r}

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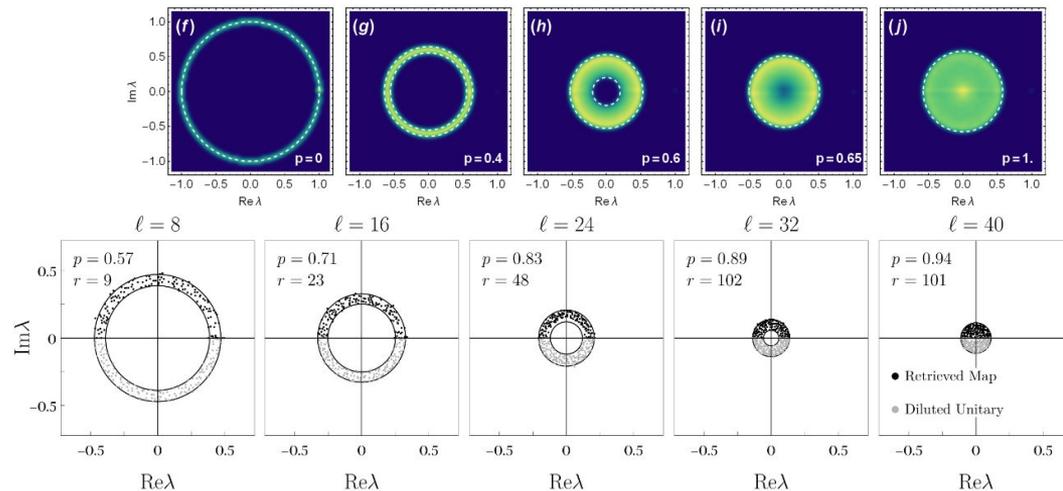
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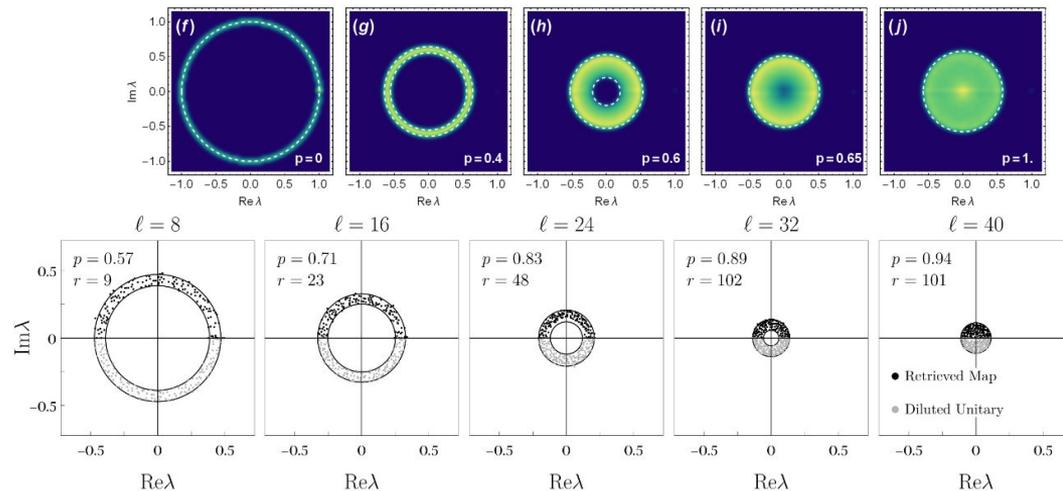
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- DU capture the spectral support of PQC. It tells us about the generic/universal behaviour of real noise.
- Also potentially exclude typical reductionistic noise models in. E.g. random benchmarking and gate-set tomography.

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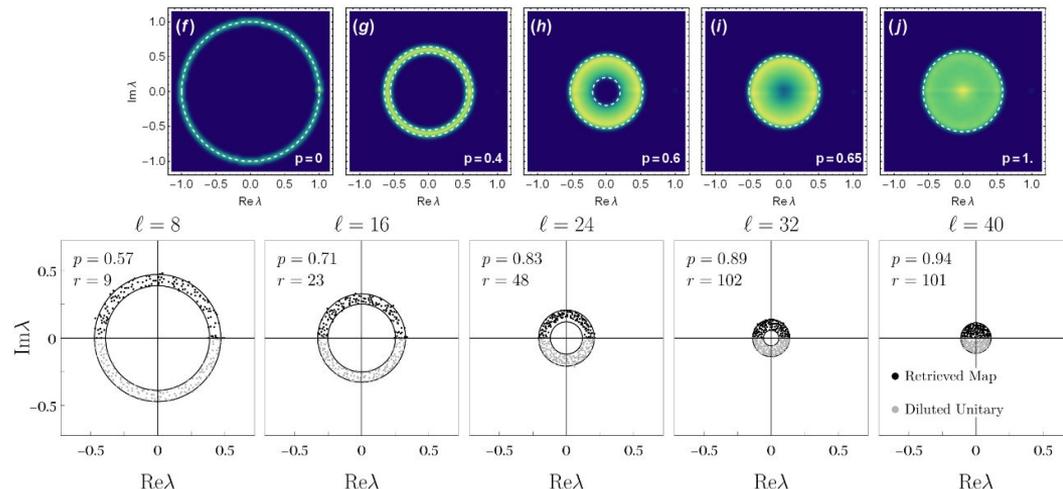
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$$R_{\pm} = \frac{1}{\sqrt{d}} \sqrt{(1-p)^2 d \pm p^2}$$

- Also potentially exclude typical reductionistic noise models in. E.g. random benchmarking and gate-set tomography.
- Is not a mechanistic model in itself. DU is not have is *actually* happening in hardware. But it might guide the search for better models.



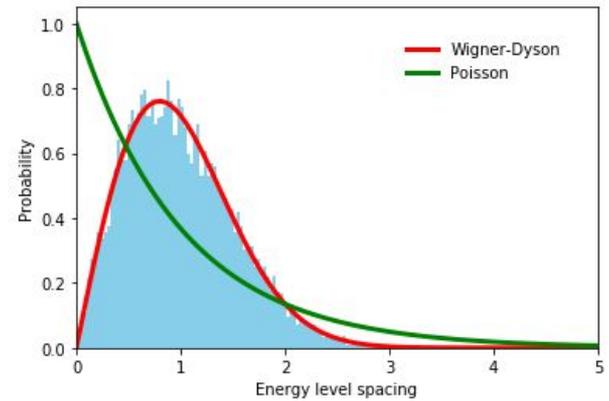
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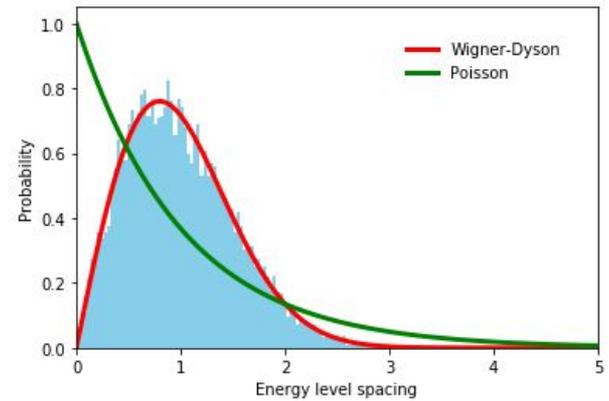
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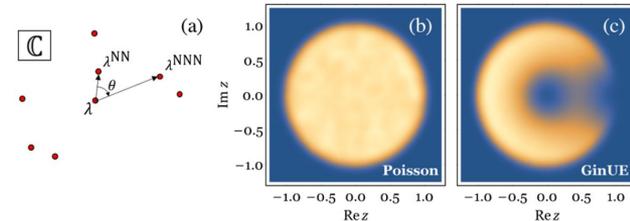
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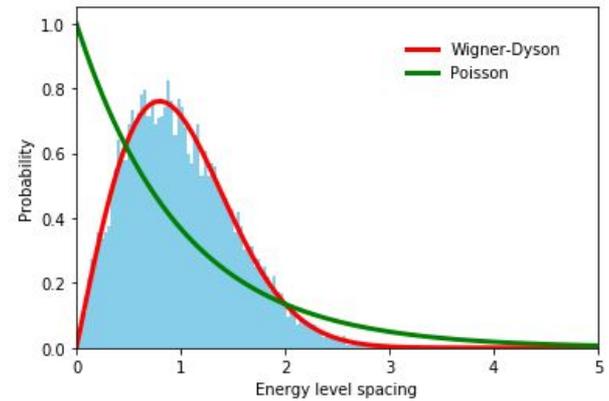
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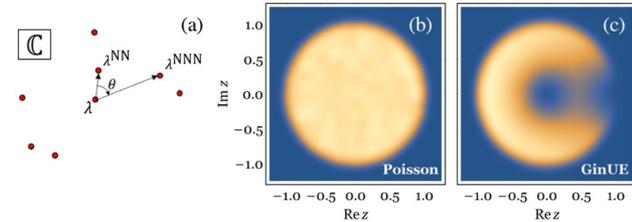
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- What systems gives statistics that are not bitten donuts, and can we even detect it accurately?



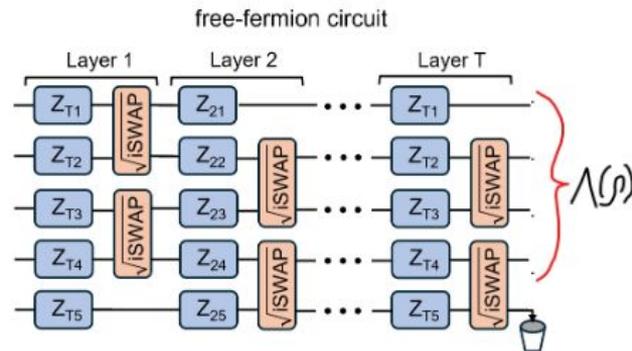
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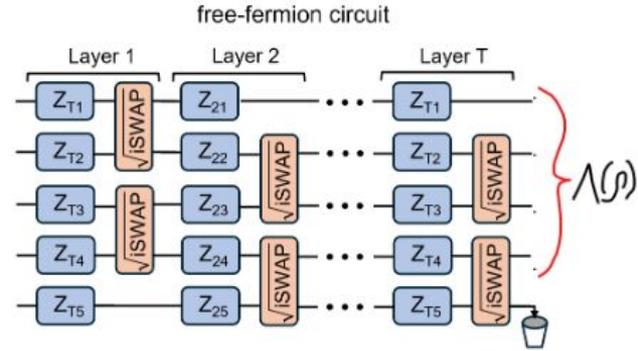
Trace-out Free-Fermion (FF) Circuits

- Inspiration from circuits simulating fermionic systems. Very constrained gate-set. Lots of symmetry.



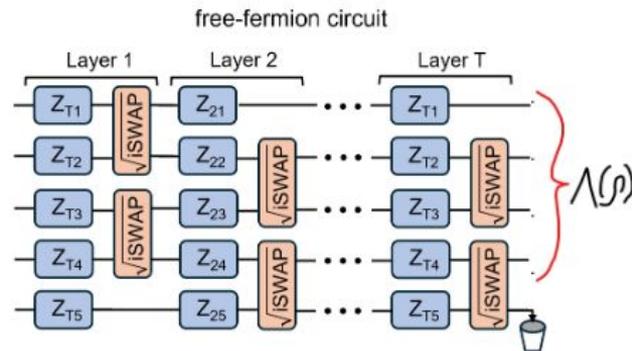
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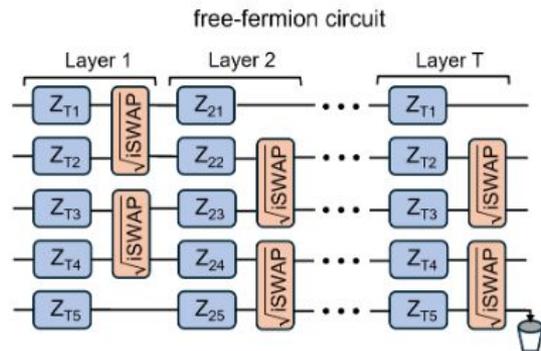
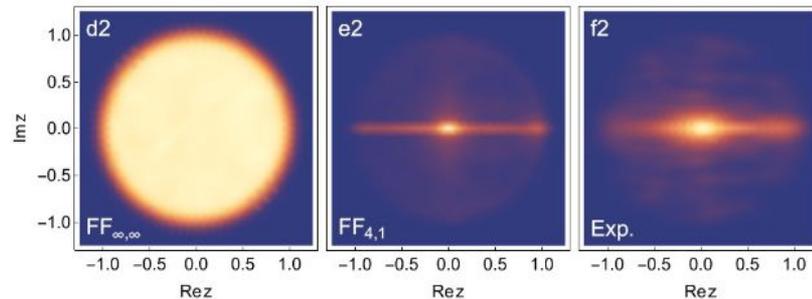
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- Inspiration from circuits simulating fermionic systems. Very constrained gate-set. Lots of symmetry.
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- For a high-fidelity computer and shallow circuits, the hope is that the engineered noise is dominating over the latent, and detectable.



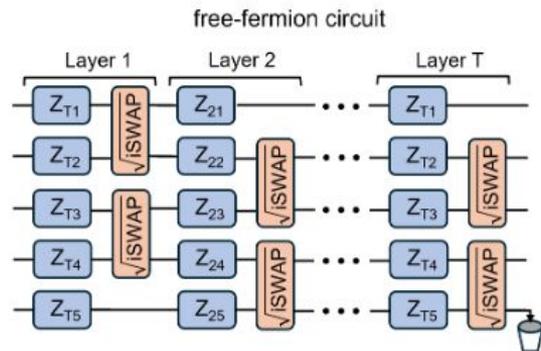
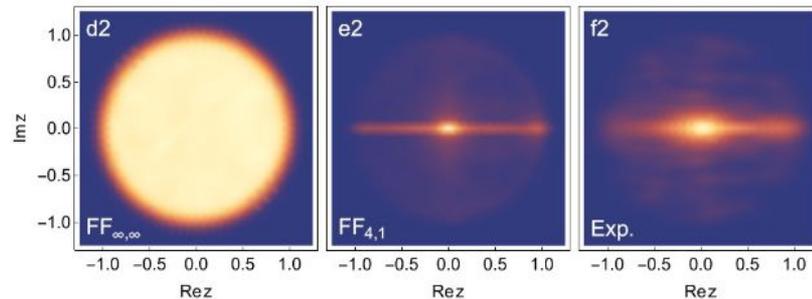
Results and Comparisons

- For the infinite size FF, we get the expected disk. For 4+1 qubits, we get strong finite size effects. But at any rate, it is very different from the bitten donut.



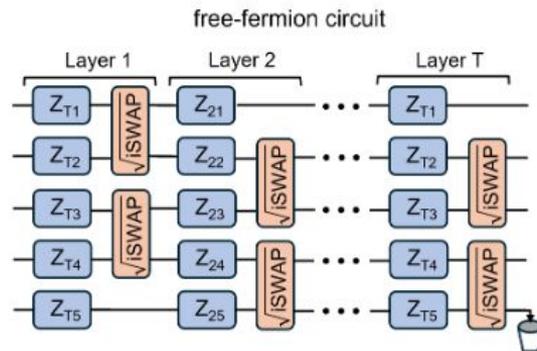
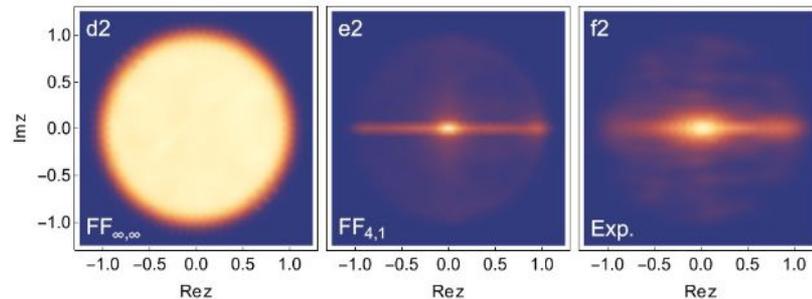
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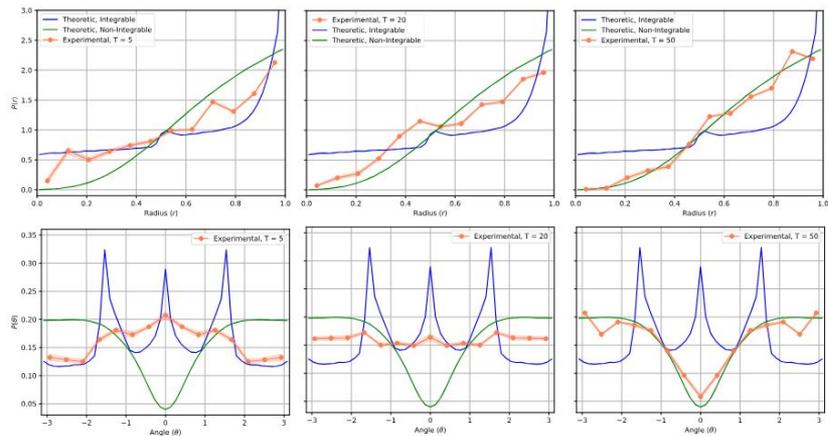
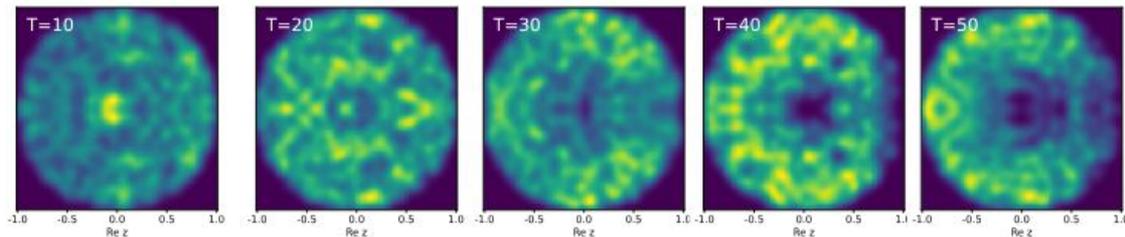
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- Previous was for $l=5$. How does the CSR change as you increase the number of layers?



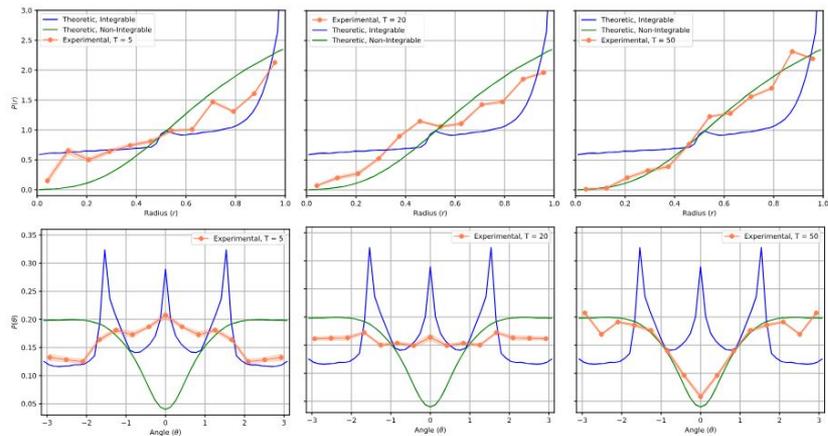
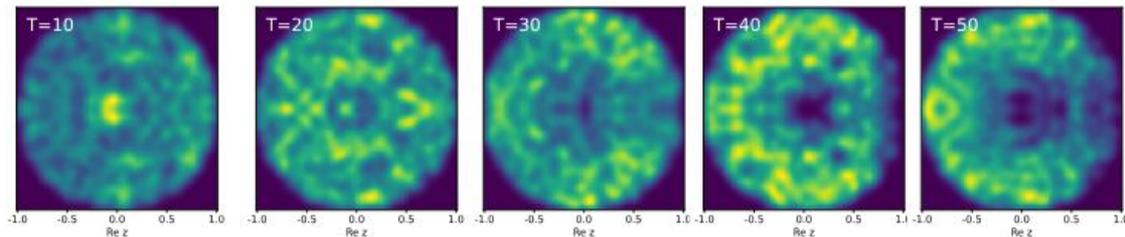
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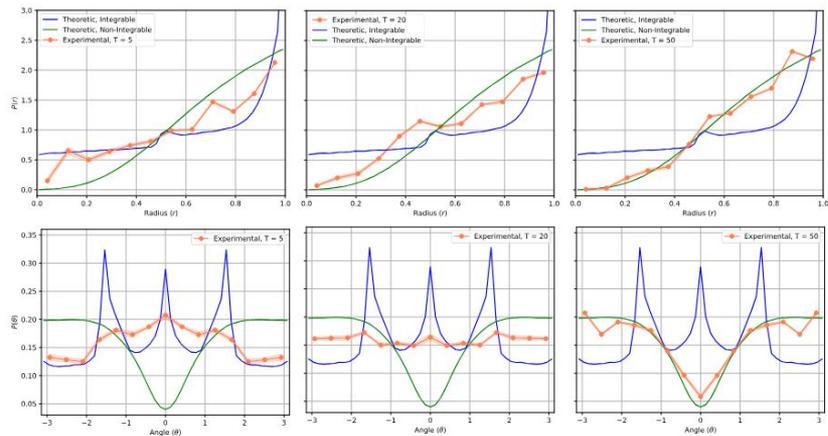
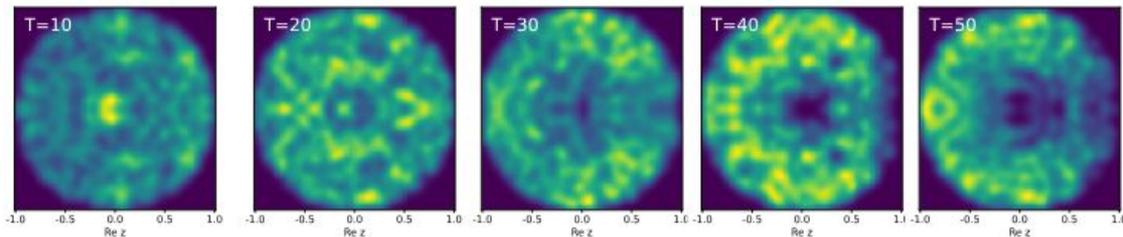
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- We see observe that the latent noise induce a disk to donut transition, showing that it is chaotic in nature.
- Further observations that hopefully inspire how to think about quantum noise.



Quantum Process Tomography

- Other presentation!